Dynamics of spiral waves under the modulation of noise pulses

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This work aims at investigating the dynamics of spiral waves under the modulation of noise pulses. Both rigid rotating and meandering spirals are considered. The numerical simulations show that for meandering spirals there exists a minimal external radius of the tip trajectory at an optimal intensity when keeping the duration constant, or for an optimal duration when keeping the intensity constant. For rigid rotating spirals an interesting phenomenon is that the clockwise-counterclockwise transition of the trajectory occurs when we raise the duration of the noise pulse for a fixed intensity.

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I. INTRODUCTION

Spiral waves not only occur in many excitable media including chemical, biological, and physical system, such as cardiac tissue, aggregating slime-mould cells and CO oxidation on platinum, but also appear in spontaneously oscillating media like the *BZ* reaction. The tip of a rotating spiral wave traces circular or hypocycloidal trajectories that depend on the medium excitability. With the change of the excitability of the medium, steadily rotating spiral waves become unstable. The transition from steady rotation to meandering motion occurs via a Hopf bifurcation [1–8].

In the last few years, the dynamics of spiral waves modulated by periodic force, pulse, or feedback has attracted many interests. All the research aims at controlling the motion of spiral waves. With the modulation of periodic sinusoidal signal, resonance drift and entrainment bands are observed in excitable media [9–13]. As to the pulsatory modulation, it is found that periodic stimulation of the medium can also produce spiral wave entrainment and resonance, but the feedback-controlled periodic pulse will introduce entrainment or resonance attractor [14–16]. Finally, the global feedback in confined geometries can stabilize the rigid rotation and suppress self-sustained activity [17].

In this work, we use the noise pulse to modulate the dynamics of spiral waves. The influences of noise on spatial patterns have been studied in some earlier works [18–21]. They often make stationary patterns become fuzzy and traveling waves break up or even restrain waves from propagating. However, noise can also play a constructive role on supporting waves in some conditions. We consider noise pulse modulating dynamics of spiral waves because noise always exists in actual systems, and has some special properties. In addition, we found that although the dynamics of meandering spirals can be controlled effectively with noise modulation, it hardly influences the dynamics of rigid rotating spirals. With the modulation of noise pulse, the numerical simulations indicate that noise pulse can not only stabilize the tip trajectories of meandering spirals and induce spatiotemporal stochastic resonance (STSR) at a suitable duration or intensity of noise pulse, but can also push rigid rotating spirals to drift along a specific direction. Especially, the rigid rotating spiral wave exhibits richer dynamics under the modulation of the noise pulse.

II. MODEL AND DYNAMICS UNDER STATIONARY CONDITIONS

The model we investigate was introduced by Barkley [22]. It is a special case of the Fitzhugh-Nagumo model, and consists of the following reaction-diffusion equations:

$$\frac{\partial u}{\partial t}=u(1-u)\left(u-\frac{v+b}{a}\right)\bigg/\ \varepsilon+\nabla^2 u, \tag{1}$$

$$\frac{\partial v}{\partial t}=u-v,$$

where a, b, and ε are parameters, $\varepsilon \ll 1$. In the equations, the b value is related to the threshold of excitation in the media. With the increase of the b value, the excitability of the medium decreases. The change of excitability will lead to the transition of the spiral dynamics from quasiperiodic meandering motion to rigid rotation, which has no hysteresis, and displays supercritical nature. The steady spiral waves (RW) rotate with a definite period T_0, but the quasiperiodic meandering spiral waves (MRW) are seen to rotate in a compound way with two periods T_1 and T_2. Here, we use T_2 to denote the primary period that can be measured far outside the rotation center of spiral wave, T_1, to denote secondary period that can be considered as the interval of absolute concentration maxima or minima [1,11]. Sometimes, the primary period T_2 is approximately treated as the average time which the tip forms a single petal of the meander flower. In fact, T_2 and T_1 are the periods of spirals rotating around the core center and the trajectory center, respectively. In addition to RW and MRW, there exist modulated traveling spiral waves (MTW) that the cores almost drift along a straight line and the period T_1 tends to infinite [1,3].

We carry out the numerical simulations on a two-dimensional lattice of 170×170 cells. The algorithm is proposed by Barkley [22]. Small calculating steps are adopted

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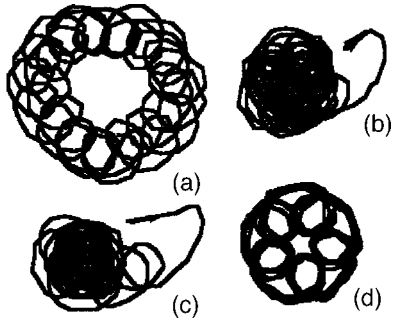

FIG. 1. Tip trajectories of MRW under the modulation of the noise pulse with intensity $D=7.0\times10^{-2}$. The parameters are $a=0.6$, $b=0.053$, $\varepsilon=0.02$, and $T_{\mathrm{mod}}=2.332$ time units. The durations of pulse are (a) $\tau=1.0$ time units, (b) $\tau=1.9$ time units, (c) $\tau=2.03$ time units, and (d) $\tau=2.13$ time units.

for keeping higher precision: the time step $dt=0.001$, the grid spacing $h=0.2$. The numerical simulation can help us learn the dynamics of spiral waves under noise modulation, such as, the tip trajectory, the spiral shape, and the curvature of the wave front. Then, a fast Fourier transform is applied to the time series of tip to obtain the periods T_1 and T_2.

III. DYNAMICS UNDER THE MODULATION OF THE NOISE PULSE

Now we consider the effect that the noise pulse imposes on spiral waves. As we know, the excitability of the media

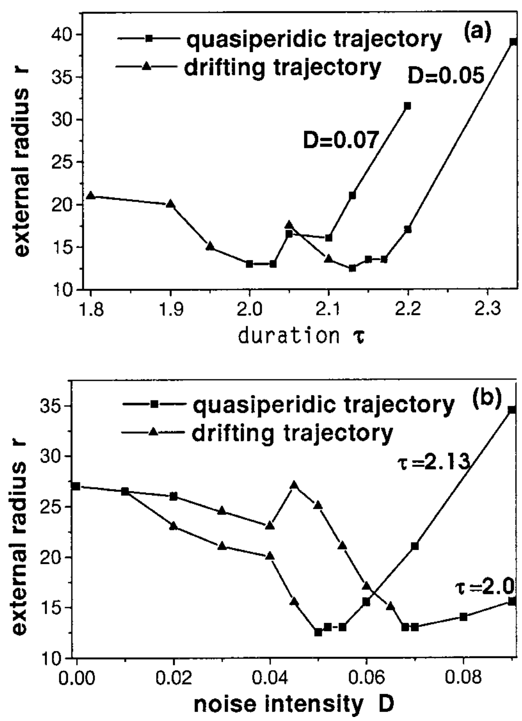

FIG. 2. The external radius of the tip trajectory is shown as a function of the duration of noise pulse and noise intensity for the medium with parameters $a=0.6$, $b=0.053$, $\varepsilon=0.02$, and $T_{\mathrm{mod}}=2.332$ time units. The other parameters are (a) the noise intensities $D=5.0\times10^{-2}$, 7.0×10^{-2}; (b) the duration of pulse $\tau=2.0$ and 2.13 time units.

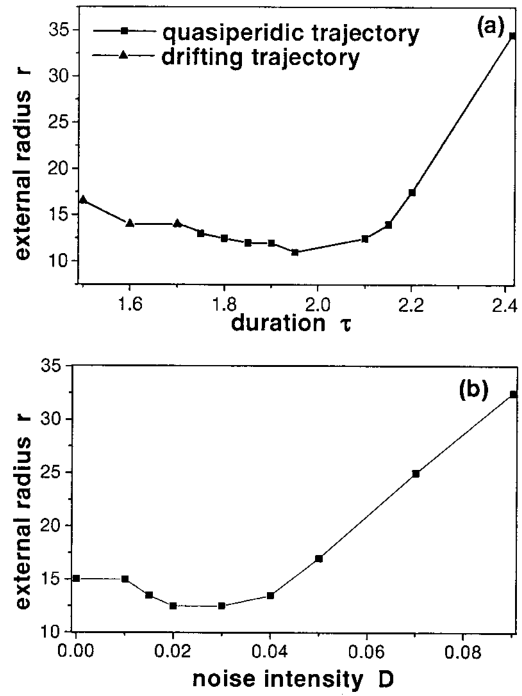

FIG. 3. The external radius of the tip trajectory is shown as a function of the duration of the noise pulse and the noise intensity for the medium with parameters $a=0.6$, $b=0.057$, $\varepsilon=0.02$, and $T_{\mathrm{mod}}=2.412$ time units. The other parameters are (a) the noise intensity $D=3.0\times10^{-2}$; (b) the duration of pulse $\tau=2.07$ time units.

can be modulated in many ways. Here, we use noise pulse to perturb parameter b that decides the threshold of excitation. It is produced by Gaussian white noise,

$$b=b_0+\xi(t), \qquad (2)$$

$$\langle\xi(t)\rangle=0 \quad \langle\xi(t)\xi(t')\rangle=2D\delta(t-t'),$$

where D is noise intensity. The period of noise pulse is denoted as T_{mod} and the duration of pulse is denoted as τ. When T_{mod} is equal to the duration of pulse, the pulse becomes white noise again.

In Refs. [15,16], it has been mentioned that an applied single impulse deformed the trajectories though they stabilized soon after stimulus. The periodic pulse with the same amplitude induces entrainment band or resonance drift in the medium. When we modulate the system with the pulse of Gaussian white noise, the situation will be much more complicated because there exist three factors relating to the modulation of the excitability. They are the modulation period T_{mod} of noise pulse, the duration of noise pulse, and the noise intensity. For the medium with parameters: $\varepsilon=0.02$, $a=0.6$, $b=0.053$, it is found that the ruleless drift of trajectory appears in the dynamics of spirals when the modulation period T_{mod} of noise pulse is larger than the period T_2 of the system, i.e., when the ratio T_{mod}/T_2 is larger than 1/1. In fact, this conclusion is also applicable to the other media. As the trajectory drift induced by noise pulse often appears ruleless, we do not consider this kind of drift as the emphasis of the present work. For $T_{\mathrm{mod}}/T_2=1/2$, the noise pulse can re-

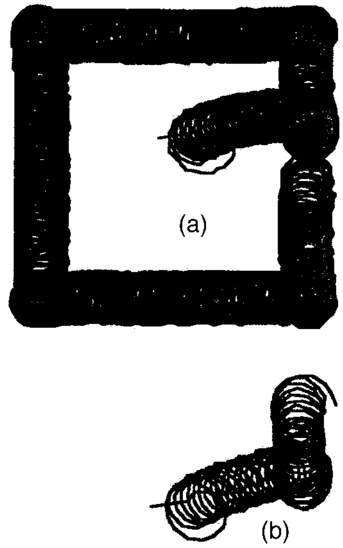

FIG. 4. Tip trajectories of RW under the modulation of the noise pulse with duration $\tau=2.72$ time units. The parameters are $a=0.6$, $b=0.068$, $\varepsilon=0.02$, and $T_{\rm mod}=3.293$ time units. The intensities of pulse are (a) $D=1.0\times10^{-2}$; (b) $D=2.0\times10^{-2}$.

duce the external radius of the trajectories under some conditions, thus improve the stability of the tip trajectories. Therefore, we discuss the situations in which the media are modulated by two ways for the ratio $T_{\rm mod}/T_2=1/2$: (a) varying the duration of pulse at constant noise intensity; (b) varying noise intensity at constant duration of pulse.

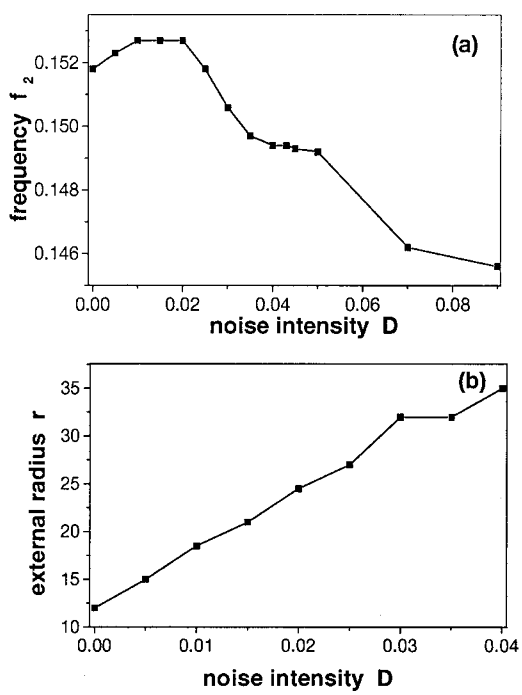

FIG. 5. The variance of rotation frequency of spiral waves and the external radius of trajectories with the intensity of noise pulse. The parameters are, $a=0.6$, $b=0.068$, $\varepsilon=0.02$, and $T_{\rm mod}=3.293$ time units. The duration is $\tau=2.842$ time units.

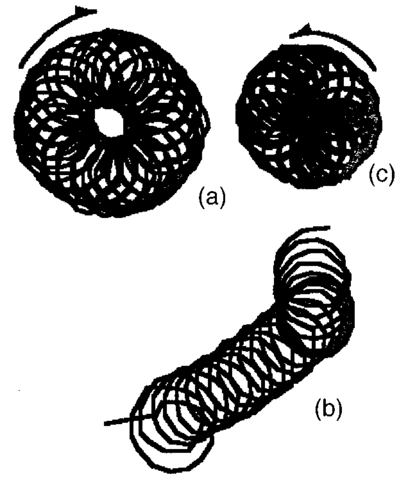

FIG. 6. Tip trajectories of RW under the modulation of the noise pulse with intensity $D=3.0\times10^{-2}$. The parameters are $a=0.6$, $b=0.068$, $\varepsilon=0.02$, and $T_{\rm mod}=3.293$ time units. The duration of pulse are (a) $\tau=2.6$ time units, (b) $\tau=2.73$ time units, and (c) $\tau=2.9$ time units.

A. Modulation of MRW

The media with parameters $\varepsilon=0.02$, $a=0.6$, $b=0.053$–0.060 exhibit MRW. Figure 1 show that at constant noise intensity $D=7.0\times10^{-2}$, the tip trajectories of MRW change with the pulse duration for the media with parameter $b=0.053$. When the pulse duration τ is smaller than 1.9 time units, the resonancelike drift occurs [Fig. 1(a)]. With the increase of the duration, the drifting trajectories begin to converge to a limited region [Fig. 1(b)]. When the duration τ reaches 2.03 time units, the spiral waves almost rotate around a circle rigidly [Fig. 1(c)]. Namely, the trajectory is stabilized by noise pulse. However, a further increase of the duration will immediately lead to the transition from RW to meandering rotation MRW. The external radius of trajectory will increase [Fig. 1(d)]. It indicates that there exists a minimum of the external radius of trajectories for an optimal duration of noise pulse.

Similarly, we can get the result that the external radius changes with the noise intensity for a fixed duration of pulse

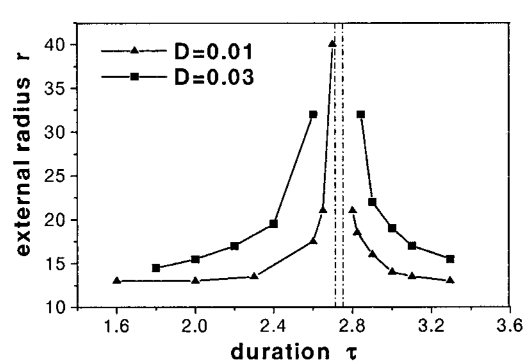

FIG. 7. The variance of the external radius of trajectories with the duration of the noise pulse. The parameters are $a=0.6$, $b=0.068$, $\varepsilon=0.02$, $T_{\rm mod}=3.293$ time units. The noise intensities are $D=1.0\times10^{-2}$, 3.0×10^{-2}.

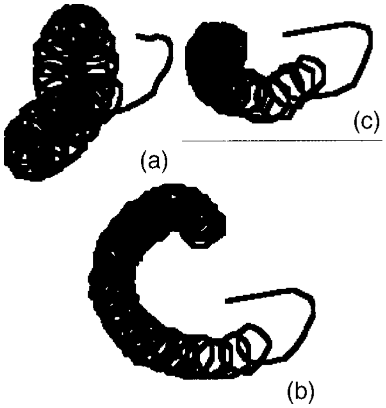

FIG. 8. Tip trajectories of MTW under the modulation of the noise pulse with intensity $D=4.0\times10^{-2}$. The parameters are $a=0.87$, $b=0.115$, $\varepsilon=0.02$, and $T_{\mathrm{mod}}=2.239$ time units. The durations of pulse are (a) $\tau=1.4$ time units, (b) $\tau=1.8$ time units, (c) $\tau=2.1$ time units.

$\tau=2.0$ or 2.13 time units. There also exists an optimal noise intensity at which the external radius of trajectory is minimum. This phenomenon indicates that noise pulses can synchronize the dynamics of spiral waves. This takes on the character of STSR.

Usually, the trajectory drift occurs for small duration or intensity of the pulse. If the duration or intensity of the noise pulse is large enough, the drifting trajectories will concentrate in a limited region. We regard one-half of the size of the region as the external radius of trajectories. Then we plot Fig. 2 to illustrate how the external radius changes with the duration or intensity of the pulse. Figure 2 indicates that the optimal duration of pulse is larger at a smaller intensity, and the optimal intensity is smaller for a longer duration. At the same time, Fig. 2(b) displays that STSR phenomenon will not take place if the duration of pulse is smaller than a critical value because the trajectory always tends to drift at the time. For the media near Hopf bifurcation the same phenomena are observed, but the optimal intensity or duration of the pulse is smaller. It means that STSR phenomenon takes place more easily in the media near Hopf bifurcation (Fig. 3).

B. Modulation of RW

Generally, the noisy environment hardly influences the tip trajectories of the rigidly rotating spiral waves unless the noise is very strong. In contrast to this, the tip of spiral traces various trajectories under the modulation of noise pulse. For a fixed duration of pulse, the drift velocity of trajectory and the rotation frequency of spiral are directly related to the intensity of noise pulse. In Fig. 4, we see that the spiral waves take 700 time units and 280 time units to reach the top right corner at the intensity of pulse $D=1.0\times10^{-2}$ and 2.0×10^{-2}, respectively. This indicates that with the increase of intensity both the drift velocity and the pitch of trajectory get larger. However, the rotation frequency of spiral decreases at the same time [Fig. 5(a)]. Except for some specific duration of the pulse, RW generally are turned into MRW. According to the numerical simulations, the increase of the intensity of noise pulse will also lead to the rising of the external radius of trajectory [Fig. 5(b)].

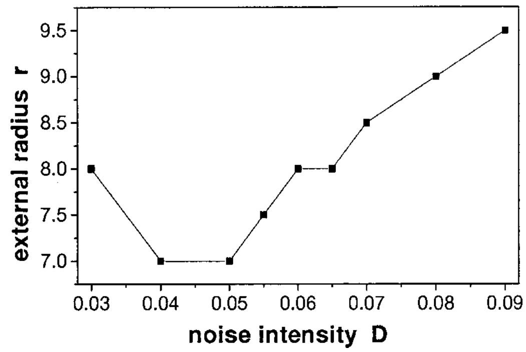

FIG. 9. The variance of the external radius of trajectories with the noise intensity for duration $\tau=2.1$ time units. The parameters are, $a=0.87$, $b=0.115$, $\varepsilon=0.02$, and $T_{\mathrm{mod}}=2.239$ time units.

When we change the duration of pulse at constant intensity, we observe an interesting phenomenon that exhibits the drift direction transition of trajectory. For example, if we keep the intensity at $D=3.0\times10^{-2}$, the trajectory moves clockwise for a small duration $\tau=2.6$ time units [Fig. 6(a)], and counterclockwise for a large duration $\tau=2.9$ time units [Fig. 6(c)]. Between the two values, the trajectory drifts along straight lines for the duration $\tau=2.72$ time units [Fig. 6(b)]. Then we know that the transition occurs on raising or reducing the duration of pulse from $\tau=2.72$ time units. In fact, this transition point is applicable comprehensively to any intensity of noise pulse, and always lies between 2.71 and 2.75 time units (Fig. 7). Furthermore, the trajectories turn when the tip comes close to a boundary [Fig. 4(a)]. However, a large intensity will lead the tip to drift out boundary directly [Fig. 4(b)]. If the duration is longer than 2.75 time units, the trajectory will drift counterclockwise. If the duration is shorter than 2.71 time units, the trajectory will move clockwise.

C. Modulation of MTW

The media with parameters $\epsilon=0.02$, $a=0.87$, and $b=0.115$ exhibit MTW. It is similar in Sec. III A. If we expect to modulate MTW with noise pulse effectively, we should choose a suitable duration. Figure 8 indicates that for a constant intensity of pulse $D=4.0\times10^{-2}$, a small duration $\tau=1.4$ time units will lead to resonancelike drift of trajectory [Fig. 8(a)], but a middle duration $\tau=1.8$ time units will hardly influence the motion posture of MTW [Fig. 8(b)]. Only for a large duration $\tau=2.1$ time units, MTW can be modulated to move quasiperiodically [Fig. 8(c)]. In fact, the tip trajectories almost drift along the original line when the duration of pulse is middle, even if we raise the level of noise pulse very high.

After choosing a large duration of pulse, we modulate the excitability of the system with noise pulse. It is observed that STSR also occurs in the dynamics behavior of MTW (Fig. 9). There exists a minimum of external radius of trajectory at an optimal noise level.

IV. CONCLUSION

In this work, we have investigated the response of spiral waves to noise pulses. For MRW and MTW we find that STSR occurs under the modulation of the noise pulse with suitable duration. There exists a minimal external radius of trajectory at an optimal intensity on keeping the duration constant or for an optimal duration on keeping the intensity constant. Generally, STSR cannot take place for small duration of pulse.

When we raise the intensity of pulse on keeping the duration constant for RW, the drift velocity and pitch of trajectory will increase, but the rotation frequency will decrease. In addition, the external radius of trajectory will rise with the intensity if RW are translated to MRW. When we vary the duration of the pulse on keeping the intensity constant, the drift direction of trajectory displays a transition. The trajectory drifts clockwise for the duration shorter than 2.71 time units, counterclockwise for the duration longer than 2.75 time units, and along straight lines for the duration between 2.71 and 2.75 time units.

On comparing our earlier work on modulating the system with noise, we conclude that the noise pulse can control RW's behavior more effectively than noise. It can not only stabilize meandering spirals in a limited region, but can also push rigidly rotating spirals to drift along a specific direction. To do this, it is important to choose a suitable duration and intensity of the noise pulse.

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